medians and altitudes

[SQA] 1. Find the equation of the median AD of triangle ABC where the coordinates of A, B and C are $(-2, 3)$, $(-3, -4)$ and $(5, 2)$ respectively. **3**
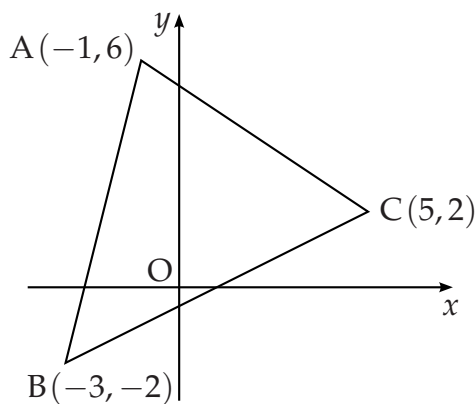
Part	Marks	Level	Calc.	Content	Answer	U1 OC1
	3	C	CN	G3, G7		1995 P1 Q5

OR

$\bullet^1$ $D = (1, -1)$

$\bullet^2$ use A and D to get $m_{AD} = -\frac{4}{3}$

$\bullet^3$ $y - 3 = -\frac{4}{3}(x - -2)$

$\bullet^1$ for showing triangle isosceles

$\bullet^2$ $m_{BC} = \frac{3}{4}$ giving $m_{AD} = -\frac{4}{3}$

$\bullet^3$ $y - 3 = -\frac{4}{3}(x - -2)$

[SQA] 2. Triangle ABC has vertices $A(-1, 6)$, $B(-3, -2)$ and $C(5, 2)$.

Find

(a) the equation of the line p, the median from C of triangle ABC. **3**

(b) the equation of the line q, the perpendicular bisector of BC. **4**

(c) the coordinates of the point of intersection of the lines p and q. **1**

Part	Marks	Level	Calc.	Content	Answer	U1 OC1
(a)	3	C	CN	G7	$y = 2$	2002 P2 Q1
(b)	4	C	CN	G7	$y = -2x + 2$	
(c)	1	C	CN	G8	$(0, 2)$	

$\bullet^1$ ss: determine midpoint coordinates
$\bullet^2$ pd: determine gradient thro' 2 pts
$\bullet^3$ ic: state equation of straight line

$\bullet^4$ ss: determine midpoint coordinates
$\bullet^5$ pd: determine gradient thro' 2 pts
$\bullet^6$ ss: determine gradient perp. to $\bullet^5$
$\bullet^7$ ic: state equation of straight line

$\bullet^8$ pd: process intersection

$\bullet^1$ $F = \text{mid}_{AB} = (-2, 2)$
$\bullet^2$ $m_{FC} = 0$ *stated or implied by* $\bullet^3$
$\bullet^3$ equ. FC is $y = 2$

$\bullet^4$ $M = \text{mid}_{BC} = (1, 0)$
$\bullet^5$ $m_{BC} = \frac{1}{2}$
$\bullet^6$ $m_\perp = -2$
$\bullet^7$ $y - 0 = -2(x - 1)$

$\bullet^8$ $(0, 2)$

3. Triangle PQR has vertex P on the x-axis, as shown in the diagram.

   Q and R are the points $(4, 6)$ and $(8, -2)$ respectively.

   The equation of PQ is $6x - 7y + 18 = 0$.
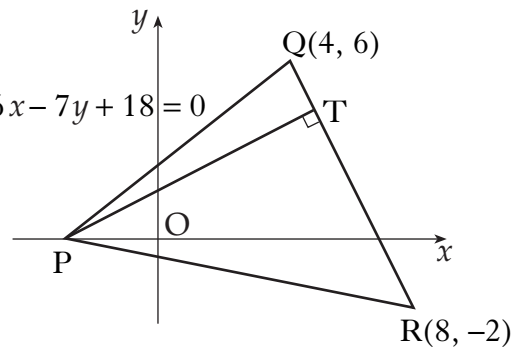
   (a) State the coordinates of P. **1**

   (b) Find the equation of the altitude of the triangle from P. **3**

   (c) The altitude from P meets the line QR at T. Find the coordinates of T. **4**

Diagram: y-axis, Q(4, 6), $6x - 7y + 18 = 0$, T, O, P, R(8, -2)

Part	Marks	Level	Calc.	Content	Answer	U1 OC1
(a)	1	C	CN	G4	$P(-3, 0)$	2009 P1 Q21
(b)	3	C	CN	G7	$y = \frac{1}{2}(x + 3)$	
(c)	4	C	CN	G8	$T(5, 4)$	

$\bullet^1$ ic: interpret x-intercept		$\bullet^1$ $P = (-3, 0)$	
$\bullet^2$ pd: find gradient (of QR)		$\bullet^2$ $m_{QR} = -2$	
$\bullet^3$ ss: know and use $m_1 m_2 = -1$		$\bullet^3$ $m_{alt.} = \frac{1}{2}$	
$\bullet^4$ ic: state equ. of altitude		$\bullet^4$ $y - 0 = \frac{1}{2}(x + 3)$	
$\bullet^5$ ic: state equ. of line (QR)		$\bullet^5$ $y + 2 = -2(x - 8)$	
$\bullet^6$ ss: prepare to solve sim. equ.		$\bullet^6$ $x - 2y = -3$ and $2x + y = 14$	
$\bullet^7$ pd: solve for x		$\bullet^7$ $x = 5$	
$\bullet^8$ pd: solve for y		$\bullet^8$ $y = 4$	

4. A quadrilateral has vertices $A(-1, 8)$, $B(7, 12)$, $C(8, 5)$ and $D(2, -3)$ as shown in the diagram.
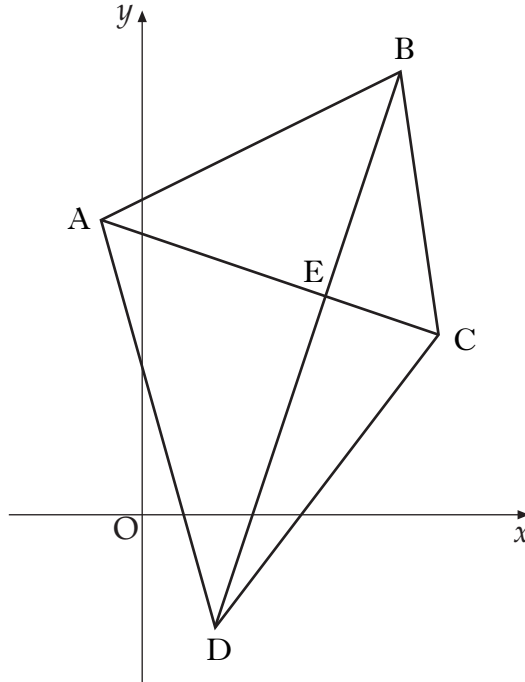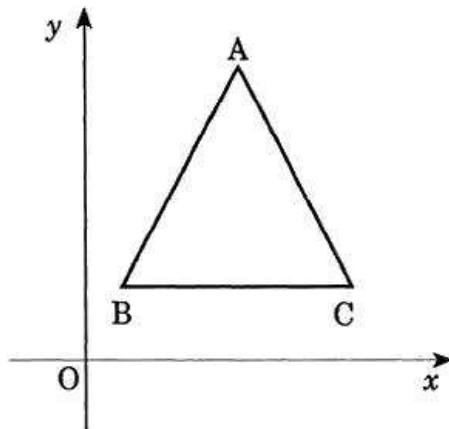

(*a*) Find the equation of diagonal BD. **2**

(*b*) The equation of diagonal AC is $x + 3y = 23$.

Find the coordinates of E, the point of intersection of the diagonals. **3**

(*c*) (i) Find the equation of the perpendicular bisector of AB.

(ii) Show that this line passes through E. **5**

Part	Marks	Level	Calc.	Content	Answer	U1 OC1
(*a*)	2	C	CN	G3, G2	$y - 12 = 3(x - 7)$	2011 P1 Q21
(*b*)	3	C	CN	G8	$E(5, 6)$	
(*ci*)	4	C	CN	G7	$y - 10 = -2(x - 3)$	
(*cii*)	1	C	CN	A6	proof	

- $\bullet^1$ pd: find gradient of BD
- $\bullet^2$ ic: state equation of BD

- $\bullet^3$ ss: start solution of simultaneous eqs
- $\bullet^4$ pd: solve for one variable
- $\bullet^5$ pd: solve for second variable

- $\bullet^6$ ss: know and find midpoint of AB
- $\bullet^7$ pd: find gradient of AB
- $\bullet^8$ ic: interpret perpendicular gradient
- $\bullet^9$ ic: state equation of perp. bisector
- $\bullet^{10}$ ic: justification of point on line

- $\bullet^1$ $\frac{15}{5}$ or equiv.
- $\bullet^2$ $y - (-3) = 3(x - 2)$

- $\bullet^3$ $3x - y = 9$ and $x + 3y = 23$
- $\bullet^4$ $x = 5$ or $y = 6$
- $\bullet^5$ $y = 6$ or $x = 5$

- $\bullet^6$ $(3, 10)$
- $\bullet^7$ $\frac{4}{8}$ or equiv.
- $\bullet^8$ $-\frac{8}{4}$ or equiv
- $\bullet^9$ $y - 10 = -2(x - 3)$
- $\bullet^{10}$ when $x = 5$, $y = -2 \times 5 + 16 = 6$

5. A triangle ABC has vertices A(4, 8), B(1, 2) and C(7, 2).

(a) Show that the triangle is isosceles. (2)

(b) (i) The altitudes AD and BE intersect at H, where D and E lie on BC and CA respectively. Find the coordinates of H. (7)

(ii) Hence show that H lies one quarter of the way up DA. (1)

Part	Marks	Level	Calc.	Content	Answer	U1 OC1
(a)	2	C	CN	G1	proof	1995 P2 Q1
(b)	8	C	CN	G8, G7, G1	(i) $H(4, \frac{7}{2})$, (ii) proof	

(a) •¹ Calculate the length of the sides

•² $AB = AC = \sqrt{3^2 + 6^2}$

(b) •³ knows to find equ. of an altitude

•⁴ $m_{AC} = -2$

•⁵ $m_{BE} = \frac{1}{2}$

•⁶ $y - 2 = \frac{1}{2}(x - 1)$

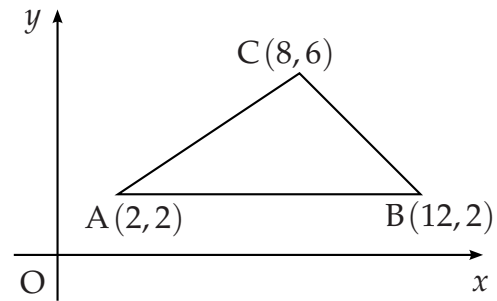•⁷ $x = 4$ stated or implied

•⁸ knows how to find intersection

•⁹ $H = \left(4, \frac{7}{2}\right)$

•¹⁰ $DA = 6$ and $DH = 1\frac{1}{2}$

6. Triangle ABC has vertices $A(2,2)$, $B(12,2)$ and $C(8,6)$.

(a) Write down the equation of l_1, the perpendicular bisector of AB. **1**

(b) Find the equation of l_2, the perpendicular bisector of AC. **4**

(c) Find the point of intersection of lines l_1 and l_2. **1**

(d) Hence find the equation of the circle passing through A, B and C. **2**

Part	Marks	Level	Calc.	Content	Answer	U2 OC4
(a)	1	C	CN	G3, G7	$x = 7$	2001 P2 Q7
(b)	4	C	CN	G7	$3x + 2y = 23$	
(c)	1	C	CN	G8	$(7,1)$	
(d)	2	A/B	CN	G8, G9, G10	$(x-7)^2 + (y-1)^2 = 26$	

$\bullet^1$ ic: state equation of a vertical line

$\bullet^2$ pd: process coord. of a midpoint
$\bullet^3$ ss: find gradient of AC
$\bullet^4$ ic: state gradient of perpendicular
$\bullet^5$ ic: state equation of straight line

$\bullet^6$ pd: find pt of intersection

$\bullet^7$ ss: use standard form of circle equ.
$\bullet^8$ ic: find radius and complete

$\bullet^1$ $x = 7$

$\bullet^2$ midpoint $= (5,4)$
$\bullet^3$ $m_{AC} = \frac{2}{3}$
$\bullet^4$ $m_\perp = -\frac{3}{2}$
$\bullet^5$ $y - 4 = -\frac{3}{2}(x - 5)$

$\bullet^6$ $x = 7, y = 1$

$\bullet^7$ $(x-7)^2 + (y-1)^2$
$\bullet^8$ $(x-7)^2 + (y-1)^2 = 26$

or

$\bullet^7$ $x^2 + y^2 - 14x - 2y + c = 0$
$\bullet^8$ $c = 24$

7. Triangle ABC has vertices $A(4,0)$, $B(4,16)$ and $C(18,20)$, as shown in the diagram opposite.

Medians AP and CR intersect at the point $T(6,12)$.
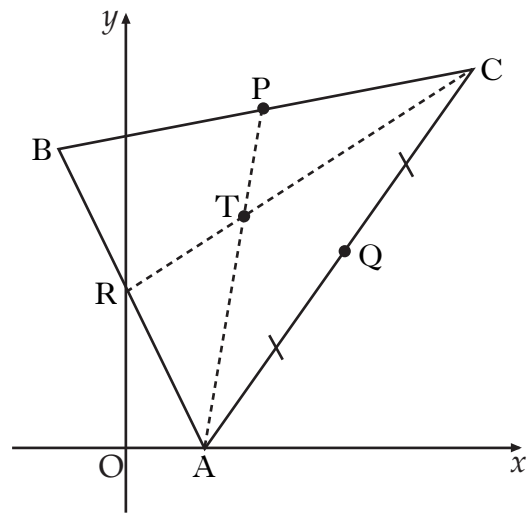
(a) Find the equation of median BQ. **3**

(b) Verify that T lies on BQ. **1**

(c) Find the ratio in which T divides BQ. **2**

Part	Marks	Level	Calc.	Content	Answer	U3 OC1
(a)	3	C	CN	G7	$y - 16 = -\frac{2}{5}(x - (-4))$	2010 P1 Q21
(b)	1	C	CN	A6	proof	
(c)	2	C	CN	G24	$2:1$	

- $\bullet^1$ ss: know and find midpoint of AC
- $\bullet^2$ pd: calculate gradient of BQ
- $\bullet^3$ ic: state equation

- $\bullet^4$ ic: substitute in for T and complete

- $\bullet^5$ ss: valid method for finding the ratio
- $\bullet^6$ ic: complete to simplified ratio

- $\bullet^1$ $(11, 10)$
- $\bullet^2$ $-\frac{6}{15}$ or equiv
- $\bullet^3$ $y - 16 = -\frac{2}{5}(x - (-4))$
  or $y - 10 = -\frac{2}{5}(x - 11)$

- $\bullet^4$ $2(6) + 5(12) = 12 + 60 = 72$

- $\bullet^5$ e.g. vector approach
  $\overrightarrow{BT} = \begin{pmatrix} 10 \\ -4 \end{pmatrix}, \overrightarrow{TQ} = \begin{pmatrix} 5 \\ -2 \end{pmatrix}$
- $\bullet^6$ $2:1$

[END OF QUESTIONS]